

一种紊流润滑理论分析新方法 ——复合型紊流模式理论

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摘 要 在对主要用于高速轻载工况的常用紊流理论进行简要分析和充分考察润滑流场的边界条件及内部结构的基础上,采用理论上比现有紊流润滑理论更为合理的复合型紊流模式理论,即在近壁区采用低紊流雷诺数的 $k-\epsilon$ 模式,而在紊流核心区采用代数雷诺应力模式,对复杂流场的紊流润滑进行了分析,同时在计入惯性效应的情况下,推导出了一种适用于高压密封和高速重载轴承等复杂流场的紊流雷诺润滑方程.利用这种复合型紊流模式理论对 Couette 型紊流流场进行了分析计算,计算结果与实验数据十分吻合,验证了模型的有效性,可以应用于高压密封和高速重载轴承之类有复杂流场的紊流润滑分析.

关键词 紊流润滑,复杂流场,惯性效应,紊流模式,代数雷诺应力模式

1 前言

随着机器向高速和大功率化发展,主要用于高速轻载工况的经典紊流润滑理论已不能满足日益增多的复杂紊流流场(如高压密封和高速重载滑动轴承等)的实用要求.早在本世纪 50 年代,Constantinescu^[1]就以 Prandtl 混合长度理论为依据建立了紊流润滑理论,但其中未计入惯性项,而且只考虑了粘性底层和完全紊流区而忽视了过渡区的存在,同时周向流和轴向流也都被人为地解偶,未计入两者之间的相互影响.显然,这种处理只能够近似地适用于以单向 Couette 流为主的高速轻载滑动轴承.60 年代相继问世的且应用相当广泛的以 Reichardt 壁面定律为依据的 Elrod-Ng^[2]和 Ng-Pan^[3]紊流润滑理论,同样也都未计入惯性项,而且后者还作了线性近似处理以简化计算.但是,由于 Reichardt 式是在单向常应力条件下得出的,尽管经过他们的人为“局部化”处理,终究还是难以解算高压密封和重载轴承之类有复杂流场的紊流润滑问题.基于经验建立起来的 Hirs 紊流润滑理论^[4],其适用范围更具有局限性,这是因为它还存在着严重的理论缺陷^[5].

以上所述均为零方程模式理论,都不敷实际应用所需,因而一些新的模式理论如一方程和两方程的模式理论等都陆续被引入紊流润滑领域.但是,由于复杂的紊流润滑流场含有一个方向的剪切流和两个方向的压力流,而且三者可属同一数量级,在膜厚方向上又有附壁区、核心区和过渡区之分,故很难用单一模式来完善地描述整个润滑流场.

因此,本文采用两种较为先进的紊流模式理论的复合,即以低紊流雷诺数 $k-\epsilon$ 理论描述过渡区,而以代数雷诺应力理论描述核心区,推导了一种更为合理的紊流雷诺润滑方程,并

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将平均惯性效应直接纳入雷诺方程.

2 基本方程

2.1 紊流模式

在现有的紊流模式中,由于两方程 k - ε 模式理论和代数雷诺应力模式理论引入了能反映紊流内部机理的紊动能传输方程和能量耗散方程,因而比其它模式方程具有更坚实、更完善的理论基础,尤其代数雷诺应力模式理论还直接计入了固体壁面对紊流脉动参数的阻滞效应. 由于润滑流场中有两个固体壁面存在,固壁效应就更为重要. 由此可见,代数雷诺应力模式理论是目前研究润滑流场比较合理的紊流模式. 但是,这种理论由于在进行模式处理简化时略去了粘性效应的影响,致使其不适用于近壁区即由粘性底层和过渡区组成的区域,而只适用于紊流核心区.

本文采用一种复合型的紊流模式理论,即在近壁区用低紊流雷诺数的 k - ε 方程,在紊流核心区用代数雷诺应力模式,两区的分界条件按紊流雷诺应力占总应力的95%来确定.

低紊流雷诺数的 k - ε 模式理论是由 Jones 等^[6]提出的,后经 Hassid 等^[7]作了改进. 本文在此采用后者(不计 Dk/Dt 和 $D\varepsilon/Dt$ 项):

$$\left. \begin{aligned} \frac{\partial}{\partial y} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right] + \nu_t \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] - \varepsilon - \frac{2\nu \cdot k}{y^2} &= 0, \\ \frac{\partial}{\partial y} \left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial y} \right] + C_{\varepsilon 1} \cdot \nu_t \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \frac{\varepsilon}{k} \\ - C_{\varepsilon 2} [1 - 0.3 \exp(-R_i^2)] \frac{\varepsilon^2}{k} - 2\nu \left(\frac{\partial \varepsilon^{1/2}}{\partial y} \right)^2 &= 0. \end{aligned} \right\} \quad (1)$$

$$\text{式中 } \nu_t = C_m [1 - 0.98 \exp(-A_m \cdot R_i)] \frac{k^2}{\varepsilon},$$

$$R_i = \frac{k^2}{\nu \cdot \varepsilon},$$

$$C_m = 0.90, \sigma_k = 1.00, \sigma_\varepsilon = 1.30, C_{\varepsilon 1} = 1.45, C_{\varepsilon 2} = 2.00.$$

对牛顿流体, $A_m = 0.0015$.

代数雷诺应力模式理论^[8]:

$$\left. \begin{aligned} \frac{\partial}{\partial y} \left[\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial y} \right] + P_k - \varepsilon &= 0, \\ \frac{\partial}{\partial y} \left[\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial y} \right] + C_{\varepsilon 1} \cdot P_\varepsilon \frac{\varepsilon}{k} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} &= 0. \end{aligned} \right\} \quad (2)$$

$$\text{式中 } \nu_t = 0.09 G_\mu \frac{k^2}{\varepsilon},$$

$$P_k = \nu_t \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right],$$

$$G_\mu = \frac{1 + \frac{3}{2} \frac{C_2 \cdot C_2}{1 - C_2} f}{1 + \frac{3}{2} \frac{C_1}{C_1} f} \cdot \frac{1 - 2 \frac{C_2 \cdot C_2 \cdot P_k / \varepsilon}{C_1 - 1 + C_2 \cdot P_k / \varepsilon} f}{1 + 2 \frac{C_1}{C_1 + P_k / \varepsilon - 1} f},$$

$$f = \frac{k^{3/2}}{C_w \cdot y \cdot \varepsilon},$$

$$C_w = 3.72, C_1 = 1.80, C_2 = 0.60, C_1 = 0.60, C_2 = 0.30, C_{\varepsilon 1} = 1.44, C_{\varepsilon 2} = 1.92,$$

$$\sigma_k = 1.00, \sigma_\varepsilon = 1.30.$$

2.2 运动方程的一般形式

对定常、稳态流场而言,有:

$$\left. \begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y}, \\ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial P}{\partial z} + \frac{1}{\rho} \frac{\partial \tau_{zy}}{\partial y}. \end{aligned} \right\} \quad (3)$$

式中 $\tau_{xy} = \mu \frac{\partial u}{\partial y} - \rho \cdot \overline{u'v'}$, $\tau_{zy} = \mu \frac{\partial v}{\partial y} - \rho \cdot \overline{w'v'}$, $-\overline{u'v'} = \nu_t \frac{\partial u}{\partial y}$, $-\overline{w'v'} = \nu_t \frac{\partial v}{\partial y}$.
 ν_t ——紊流扩散系数.

2.3 用修正平均法对惯性项进行处理

现有的紊流润滑理论一般不计惯性项的影响,但随着机器向高速重载化发展,惯性效应不可忽略.通常,考虑惯性效应的方法大致有:小参数摄动法、流函数法、Constantinescu 平均法、直接解法和修正平均法.本文采用的是修正平均法,但处理方法却与之不同,将惯性项直接引入封闭的解析润滑方程.即取:

$$\left. \begin{aligned} \bar{f}(x, z) &= \frac{1}{h} \int_0^h \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] dy = \frac{1}{h} \left[\frac{\partial I_{xx}}{\partial x} + \frac{\partial I_{xz}}{\partial z} \right], \\ \bar{g}(x, z) &= \frac{1}{h} \int_0^h \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] dy = \frac{1}{h} \left[\frac{\partial I_{zx}}{\partial x} + \frac{\partial I_{zz}}{\partial z} \right]. \end{aligned} \right\} \quad (4)$$

式中 $I_{xx} = \int_0^h u^2 dy$, $I_{xz} = \int_0^h u \cdot w dy$, $I_{zz} = \int_0^h w^2 dy$.

令等效压力梯度为:

$$\frac{\partial P^1}{\partial x} = \frac{\partial P}{\partial x} + \rho \cdot \bar{f}, \quad \frac{\partial P^1}{\partial z} = \frac{\partial P}{\partial z} + \rho \cdot \bar{g}. \quad (5)$$

将式(4)和式(5)代入运动方程(3),得:

$$\frac{\partial P^1}{\partial x} = \frac{\partial \tau_{xy}}{\partial y}, \quad \frac{\partial P^1}{\partial z} = \frac{\partial \tau_{zy}}{\partial y}. \quad (6)$$

由式(6)可见,利用修正平均法对惯性项近似处理后得到的运动方程与不计惯性项时的运动方程相似,只不过将压力梯度 $\partial P/\partial x$ 和 $\partial P/\partial z$ 换成了等效压力梯度 $\partial P^1/\partial x$ 和 $\partial P^1/\partial z$.因此,不考虑惯性项的运动方程的求解过程及有关结果,可以引用于对惯性项进行修正平均处理后式(6)的求解,这是本文采用修正平均法考虑惯性效应的目的之一.

2.4 “雷诺”型润滑方程的推导

联立求解运动方程(6)和连续方程,并计入有关边界条件得含惯性项的“雷诺”方程:

$$\frac{\partial}{\partial x} \left(G \frac{\partial P^1}{\partial x} \right) = \frac{\partial}{\partial z} \left(G \frac{\partial P^1}{\partial z} \right) = U \frac{\partial}{\partial x} (F \cdot h) \quad (7)$$

或 $\frac{\partial}{\partial x} \left[G \left(\frac{\partial P}{\partial x} + \rho \cdot \bar{f} \right) \right] + \frac{\partial}{\partial z} \left[G \left(\frac{\partial P}{\partial z} + \rho \cdot \bar{g} \right) \right] = U \frac{\partial}{\partial x} (F \cdot h)$.

式中 $G = \frac{A_2}{A_1} B_1 - B_2$, $A_1 = \int_0^h \frac{dy}{\nu + \nu_t}$, $B_1 = \int_0^h \int_0^y \frac{dy dy}{\nu + \nu_t}$,
 $F = 1 - \frac{B_1}{A_1 \cdot h}$, $A_2 = \int_0^h \frac{y dy}{\rho(\nu + \nu_t)}$, $B_2 = \int_0^h \int_0^y \frac{y dy dy}{\rho(\nu + \nu_t)}$.

3 计算结果与分析

利用本文提出的复合型紊流模式理论对图1所示两平板之间的速度场进行了分析计

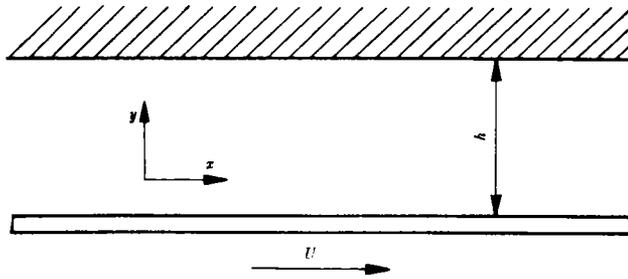


Fig. 1 Flow between two parallel plates
图1 两平板之间流场的简明示意图

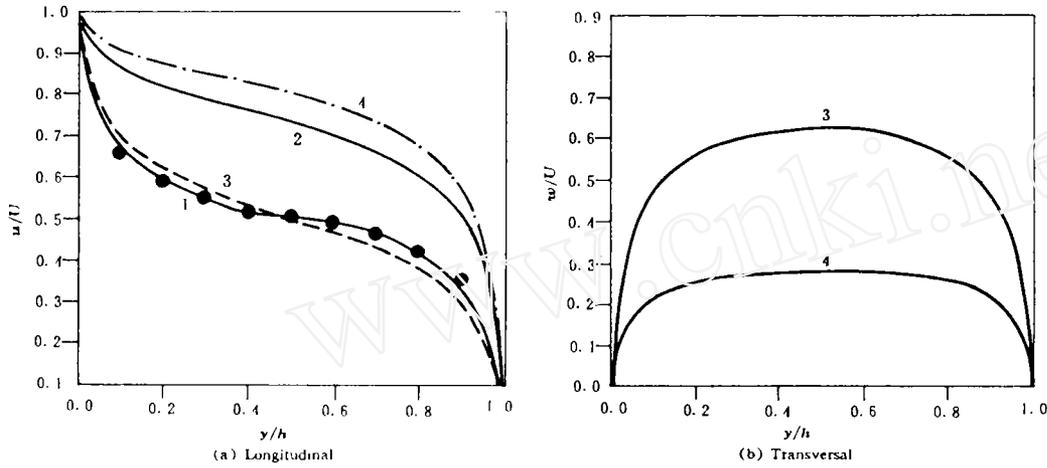


Fig. 2 Calculated results of velocity distribution ($R_s = 5\ 408$)

1. $B_x = B_z = 0$, 2. $B_x = 10, B_z = 0$, 3. $B_x = 0, B_z = 20$, 4. $B_x = 10, B_z = 20$.

Note: ● mean test value in reference [9], $B_x = B_z = 0$.

图2 纵向速度分布 ($R_s = 5\ 408$)

1. $B_x = B_z = 0$, 2. $B_x = 10, B_z = 0$, 3. $B_x = 0, B_z = 20$, 4. $B_x = 10, B_z = 20$.

注: ● 表示 Leutheusser 等^[9]所获得的实验结果, $B_x = B_z = 0$.

算, 所得结果如图 2 所示. 曲线(1)表示平面 Couette 紊流 ($R_s = 5\ 408, B_x = B_z = 0$) 的速度分布, 并与 Leutheusser 等^[9]的相应实验结果进行了比较. 由图 2 可以看出, 计算结果与实验结果吻合得很好, 表明本文采用的紊流分析法是合理可靠的. 曲线(3)和(4)分别表示有轴向压力梯度 B_z 时速度 u 和 w 的分布规律. 分别比较曲线(1)与(3)及曲线(2)与(4)可以发现, 轴向压力梯度所导致的轴向流动对周向速度分布的影响不容忽视, 尤其在有大的压力流动时更是如此. 但是, 在目前常用的紊流润滑理论中, 由于采用线性化解耦而忽略了两方向流动之间的相互影响, 这显然有其局限性. 因此, 对高压密封和高速重载轴承之类有复杂流动的场所, 采用本文提出的复合型模式进行分析更为合理.

4 结论

- a. 通过对紊流润滑膜各个分区流动特点的分析, 提出了一种复合型的紊流模式理

论. 利用这种理论能够比较充分地表达紊流润滑膜的流动特点.

b. 推导出了适用于复杂流场的紊流雷诺方程, 并且将惯性项的影响直接引入其中.

c. 利用复合型紊流模式理论对平面 Couette 流场进行了分析计算, 结果与实验值非常吻合, 表明利用本文提出的方法对复杂流场进行分析是合理可靠的.

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A New Method of Theoretical Analysis of Turbulent Lubrication Using a Combined Model of Turbulence

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Abstract Theoretical bases of the conventional turbulent lubrication theories used for high speed and light load condition are briefly analyzed. A theoretically sounder combined model of turbulent lubrication is formed, based on consideration of boundary conditions and internal structure of the lubrication flow field. The low Reynolds turbulence number $k-\epsilon$ model is used for the near-wall region, and the algebraic Reynolds stress model used for the turbulence kernel. Complex turbulent lubrication cases are analyzed by this model. A turbulent lubrication equation of the Reynolds type applicable to high pressure annular seals and heavily loaded journal bearings is derived with consideration of inertia effect. Computational results of turbulent Couette flow obtained by applying this combined turbulent lubrication model compare well with experimental data, and validity of the theory is confirmed. It can be used for turbulent lubrication analysis under high pressure annular seals and high speed-heavily loaded journal bearings.

Key words turbulent lubrication, complicated flow field, inertia effect, turbulence model, algebraic Reynolds stress model